# TESTING PROPORTIONATE HYPOTHESIS

*By*

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SUMMARY

**A proportionate hypothesis is formulated. It has been shown that the treatment sum of squares obtained under the hypothesis in Case I for the data collected from a R.B.D. has the resemblance with the treatment sum of squares computed under the null hypothesis in presence of a con-commitant variate, but the degrees of freedom in the two cases do not agree. The degrees of freedom for the treatment sum of squares in the case of proportionate hypothesis is reduced by one but the degrees of freedom for error remains unaffected.**

## 1. INTRODUCTION

Normally for drawing inference from data collected, using different experimental designs a null hypothesis of the form

$$t_1 = t_2 = \ldots t_v,$$

is made, where $t_1 = t_2, \ldots = t_v$ are the effects of the treatments. But there are situations where other types of hypotheses are more meaningful. For example, while discussing the analysis of quali-tative-cum-quantitative experiments, Yates (1937) suggested some type of proportionate hypothesis. In the present investigation we have attempted to formulate such hypothesis and provide method for their testing.

## 2. METHOD

Let $t_1, t_2, \ldots, t_v$ be the effects of $v$ treetments and $z_1, z_2, \ldots, z_v$ be $v$ known quantities. Then we form the following hypothesis

$$\frac{t_1}{Z_1} = \frac{t_2}{Z_2} \ldots = \frac{t_v}{Z_v} = t \text{ (say)},$$

where $t$ is a non-zero constant which may be known or unknown.

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This type of hypothesis has been called proportionate hypothesis and $t$ is called the constant of proportionality. For example proportionate hypothesis may be appropriate,

(*i*) When there are three non-zero doses of a fertilizers such that each dose is double its previous dose. Here the hypothesis of the type

$$\frac{t_1}{1} = \frac{t_2}{2} = \frac{t_3}{4}$$

may be made.

(*ii*) When we deal with single, double and triple gene dwarf varieties. Here a hypothesis of the form

$$\frac{t_1}{1} = \frac{t_2}{1.5} = \frac{t_3}{2}$$

may be set up.

The method will be developed with reference to data collected from R.B. designs. For obtaining the sum of squares due to error and proportionate hypothesis with their respective degrees of freedom, the method based on general linear hypothesis theory has been followed. The method in general consists of three steps :

(*i*) The model of the design concerned is stated and the residual sum of squares after fitting all the unknown constants is obtained. The residual sum of squares so obtained provides the error sum of squares and its degree of freedom is obtained by subtracting the number of independent parameters estimated from the total number of observations.

(*ii*) The second step is to modify the model according to the proportionate hypothesis. The modified model contains less number of parameters than the number of parameters in the original model. The residual sum of squares after fitting these reduced number of parameters will be denoted by Ep. S.S. and will cumprise of two portions *i.e.* sum of squares due to error and sum of squares due to proportionate hypothesis. The d.f. for Ep. S.S. is equal to total number of observations minus the number of independent normal equations for fitting the constants in the reduced model.

(*iii*) Finally, the sum of squarers due to proportionate hypothesis will be obtained by subtracting E.S.S. from Ep.S.S. The d.f. for sum of squares due to proportionate hypothesis is obtained by subtracting the d.f. for E.S.S. from the d.f. of Ep.S.S.

The details of the procedure of analysis have been discussed with reference to randomized complete block design.

3. Analysis of randomized block design :

The usual model for R.B.D. is

$$y_{ij}=\mu+b_i+t_j+e_{ij}, \qquad \ldots(1)$$

where, $y_{ij}$ is the yield of $j^{th}$ treatment in the $i^{th}$ block,

$\mu$ is the general effect,

$b_i$ is the effect of the $i^{th}$ block $(i=1,2,\ldots,r)$

$t_j$ is the effect of $j^{th}$ treatment $(j=1,2,\ldots,v)$

and $e_{ij}$'s are independent normal variates with mean zero and variance $\sigma^2$ (unknown).

By adopting the usual technique of analysis of variance and imposing the condition $\sum t_i=0$, we get the error sum of squares as

$$\text{E.S.S.}=\sum_{i,j} Y_{ij}^2-\frac{Y_{oo}^2}{rv}\left[\sum_i \frac{Y_{io}^2}{v}-\frac{Y_{oo}^2}{rv}\right]-\left[\sum_j \frac{Y_{oj}^2}{r}-\frac{Y_{oo}^2}{rv}\right] \qquad \ldots(2)$$

where, $Y_{io}=\sum_{j=1}^{v} y_{ij}$ i.e. total yield of the $i^{th}$ block,

$Y_{oj}=\sum_{i=1}^{r} y_{ij}$ i.e. total yield of the $j^{th}$ treatment,

$Y_{oo}=\sum_i \sum_j y_{ij}$ i.e. grand total of all yields.

The sum of squares due to proportionate hypothesis is obtained as below. We set up the hypothesis.

Case I : $\frac{t_1}{Z_1}=\frac{t_2}{Z_2}=\frac{t_v}{Z_v}=t$ (unknown) and find the residual sum of squares due to this hypothesis after writing $t_j=t.Z_j$ as below.

Ep.S.S.=minimum of

$$\sum_{i,j}[Y_{ij}-\mu-b_i-t\,Z_j]^2$$

with respect to $\mu$, $b_i$'s and $t$.

$$=\sum_{i,j} y_{ij}^2-\frac{Y_{oo}^2}{rv}-\left[\sum_i \frac{Y_{io}^2}{v}-\frac{Y_{oo}^2}{rv}\right]$$

$$-\frac{[\sum_j Z_jY_{oj}-(\sum_j Z_jY_{oo})/v]^2}{r[\sum Z_j^2-(\sum Z_j)^2/v]}$$

Subtracting the E.S.S, from Ep.S.S. we get treatment S.S. as,

$$\text{Tr.S.S.} = \left[\sum_j \frac{Y_{oj}^2}{r} - \frac{Y_{oo}^2}{rv}\right] - \frac{[\sum_j Z_j Y_{oj} - (\sum_j Z_j Y_{oo})/v]^2}{r\,[\sum_j Z_j^2 - (\sum_j Z_j)^2/v]} \qquad ..(3)$$

Thus, treatment sum of squares under proportionate hypothesis =Tr.S.S. under null hypothesis

$$-\frac{[\text{total sum of products of } Z \text{ and } Y]^2}{\text{total sum of squares of } Z}$$

Thus the computation of Ep.S.S. has the resemblance with the computation of Eo.S S. (treatment sum of squares under Ho treating $Z$ as a concommitant variate). It will be seen that under proportionate hypothesis E.S.S. has got $(r-1)(v-1)$ and Ep.S.S. has got $(rv-r-1)$ d.f. Hence the d.f. for the treatment S.S. is the difference of the two degrees of freedom *i,e.* $(v-2)$. One d.f. of the treatment sum of squares has been lost for estimating '$t$', the constant of proportionality. If, however, $t$ is known, one d.f. for estimating $t$ is not lost. For example if $t=0$, the proportionate hypothesis becomes the null hypothesis and no extra d.f. for treatment S.S. is lost. In this case the treatment sum of squres is

$$\left[\sum_j \frac{Y_{oj}^2}{v} - \frac{Y_{oo}^2}{rv}\right].$$

**Case II :** When $t \neq 0$ (but known), the normal equations under proportionate hypothesis becomes

$$rv\hat{\mu} + v\sum_i \hat{b}_i + Z_{oo} = Y_{oo} \qquad ...(4)$$

$$v\hat{\mu} + v\hat{b}_i + t\,Z_{io} = y_{io} \qquad ...(5)$$

$$\text{giving } (\hat{\mu} + \hat{b}_i) = \frac{Y_{io}}{v} - \frac{tZ_{io}}{v}$$

$$= \frac{Y_{io}}{v} - t(\sum_j Z_j)/v$$

The residual sum of squares under the above mentioned hypothesis is given by

$$\text{Ep.S.S.} = \sum_{i,j} y_{ij}^2 - \sum_i \frac{Y_{io}^2}{v} + rt^2 \left[ \sum_j Z_j^2 - \frac{(\sum_j Z_j)^2}{rv} \right]$$

$$-2t \left[ \sum_j Z_j Y_{oj} - \frac{Y_{oo} \sum Z_j}{v} \right], \qquad \ldots(6)$$

with $(rv-r)$ d.f.

Now the treatment sum of squares is given by

Tr.S.S. = (Ep.S.S.$-$E.S.S.) with $(v-l)$ d.f.

$$\text{or Treatment S.S.} = \left[ \sum_j \frac{Y_{oj}^2}{r} - \frac{Y_{oo}^2}{rv} \right] - 2t \left[ \sum_j Z_j Y_{oj} - \frac{Y_{oo} \sum Z_j}{v} \right]$$

$$+rt^2 \left[ \sum_j Z_j^2 - \frac{(\sum_j Z_j)^2}{v} \right], \qquad \ldots(7)$$

with $(v-l)$ d.f.

If the proportionate hypothesis in Case I is not rejected, then $t$ is estimated by

$$t = \left[ \sum_j Z_j Y_{oj} - Y_{oo} \bar{Z} \right] / r \left[ \sum_j Z_j^2 - \frac{(\sum_j Z_j)^2}{v} \right], \text{ where } \bar{Z} = \frac{l}{v} \sum_j Z_j$$

but the S.E. of $(t_j' - t_j)$ cannot be estimated when proportionate hypothesis is rejected, while the case is just reverse under the null hypothesis.

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## REFERENCE

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